A weighted Wirtinger inequality

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Abstract

We establish a sharp estimate for the best constant $C > 0$ in the Wirtinger type inequality

$$\int_0^{2\pi} aw^2 \quad \leq \quad C \int_0^{2\pi} \frac{1}{a} w'^2,$$

wheere $w \in W^{1,1}(0, 2\pi)$ is 2π-periodic and satisfies the constraint

$$\int_0^{2\pi} aw \quad = 0$$

when the non negative function $a = a(t)$ belongs to $L^1(0, 2\pi)$, extending a result of [2], [3].

We establish, also, an estimate for the best constant $C(a, b)$ in more general Wirtinger inequality

$$\int_0^{2\pi} aw^2 \quad \leq \quad C(a,b) \int_0^{2\pi} bw'^2,$$

wher w and a verify the previous assumptions and b is a non negative function such that $1/b$ and $\sqrt{a/b}$ belong to $L^1(0, 2\pi)$

[2] T. Ricciardi: “A sharp weighted Wirtinger inequality”, *Boll. Unione Mat. Ital. Sez. B Art. Ric. Mat* (8) 8 (2005), 259–267.

[3] T. Ricciardi: “A sharp Hölder estimate for elliptic equations in two variables”, *Proceedings of the Royal Society of Edinburgh* 135 A (2005), 165–173.